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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

350. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

Solve the equations:

$$\begin{aligned}x+y+z &= a_0, \\x+yu+zu &= a_1, \\x+yu^2+zu^2 &= a_2, \\x+yu^3+zu^3 &= a_3, \\x+yu^4+zu^4 &= a_4.\end{aligned}$$

Solution by A. H. HOLMES, Brunswick, Maine, and the PROPOSER.

$$\begin{aligned}x+y+z &= a_0 \dots (1); \quad x+yu+zu = a_1 \dots (2); \quad x+yu^2+zu^2 = a_2 \dots (3); \\x+yu^3+zu^3 &= a_3 \dots (4); \quad \text{and } x+yu^4+zu^4 = a_4 \dots (5).\end{aligned}$$

Subtracting (1) from (2), (2) from (3), (3) from (4), and (4) from (5), we have,

$$\begin{aligned}y(u-1)+z(v-1) &= a_1-a_0 \dots (6); \\yu(u-1)+zu(v-1) &= a_2-a_1 \dots (7); \\yu^2(u-1)+zu^2(v-1) &= a_3-a_2 \dots (8); \text{ and} \\yu^3(u-1)+zu^3(v-1) &= a_4-a_3 \dots (9).\end{aligned}$$

Eliminating y from (6), (7), (8), and (9), we have,

$$\begin{aligned}(a_1-a_0)u-zu(v-1) &= a_2-a_1-zv(v-1) \dots (10), \\(a_2-a_1)u-zuv(v-1) &= a_3-a_2-zv^2(v-1) \dots (11), \text{ and} \\(a_3-a_2)u-zuv^2(v-1) &= a_4-a_3-zv^3(v-1) \dots (12).\end{aligned}$$

Eliminating z from (10), (11), and (12), we have,

$$\begin{aligned}(a_1-a_0)uv-(a_2-a_1)v &= (a_2-a_1)u-(a_3-a_2) \dots (13), \text{ and} \\(a_2-a_1)uv-(a_3-a_2)v &= (a_3-a_2)u-(a_4-a_3) \dots (14).\end{aligned}$$

Eliminating v from (13) and (14), we have

$$\frac{(a_3-a_2)u-(a_4-a_3)}{(a_2-a_1)u-(a_3-a_2)} = \frac{(a_2-a_1)u-(a_3-a_2)}{(a_1-a_0)u-(a_2-a_1)}.$$

Putting $a_1-a_0=d_1$, $a_2-a_1=d_2$, $a_3-a_2=d_3$, $a_4-a_3=d_4$, and solving,

$$u = \frac{d_1 d_4 - d_2 d_3 \pm \sqrt{[(d_1 d_4 - d_2 d_3)^2 + 4(d_1 d_3 - d_2^2)(d_2^2 - d_2 d_4)]}}{2(d_1 d_3 - d_2^2)}.$$

From (13) we find v . Then from (10) we find z . From (6), y , and from (1), x .

351. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

$$\begin{aligned} \text{Solve, } y^2 + yz + z^2 &= a^2 \dots (1). \\ z^2 + zx + x^2 &= b^2 \dots (2). \\ x^2 + xy + y^2 &= c^2 \dots (3). \end{aligned}$$

I. Solution by J. A. COLSON, Searsport, Maine.

$$\begin{aligned} b^2 - c^2 &= (z - y)(x + y + z). \quad \therefore (b^2 - c^2)x = (zx - xy)(x + y + z). \\ c^2 - a^2 &= (x - z)(x + y + z). \quad \therefore (c^2 - a^2)y = (xy - yz)(x + y + z). \\ a^2 - b^2 &= (y - x)(x + y + z). \quad \therefore (a^2 - b^2)z = (yz - zx)(x + y + z). \\ \therefore (b^2 - c^2)x + (c^2 - a^2)y + (a^2 - b^2)z &= 0. \end{aligned}$$

For convenience, put $b^2 - c^2 = f$, $c^2 - a^2 = g$, and $a^2 - b^2 = h$. Then $f + g + h = 0$, and $fx + gy + hz = 0$.

$$\therefore z = -\frac{fx + gy}{h}, \text{ and } x + y + z = x + y - \frac{fx + gy}{h} = \frac{(h - f)x + (h - g)y}{h}.$$

$$\therefore a^2 - b^2 = h = (y - x)(x + y + z) = (y - x) \frac{(h - f)x + (h - g)y}{h}.$$

$$\therefore h^2 = (f - h)x^2 + (g - f)xy + (h - g)y^2.$$

But from (3) we have $y^2 = c^2 - x^2 - xy$.

Hence, $h^2 = c^2(h - g) + (f + g - 2h)x^2 + (2g - f - h)xy = c^2(h - g) - 3hx^2 + 3gxy$.

$$\therefore y = \frac{3hx^2 + h^2 + c^2(g - h)}{3gx}.$$

Substitute in (3), and we have

$$x^2 + \frac{3hx^2 + h^2 + c^2(g - h)}{3g} + \frac{[3hx^2 + h^2 + c^2(g - h)]^2}{9g^2x^2} - c^2 = 0.$$

Hence, clearing of fractions and uniting, we have,

$$\begin{aligned} 9(g^2 + gh + h^2)x^4 - 3[c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)]x^2 \\ + [h^2 + c^2(g - h)]^2 = 0. \end{aligned}$$

$$\begin{aligned} \therefore 36(g^2 + gh + h^2)^2x^4 - 12(g^2 + gh + h^2)[c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)]x^2 \\ + [c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)]^2 = [c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)]^2 - 4(g^2 \\ + gh + h^2)[h^2 - c^2(g - h)]^2 = 9c^4g^2h^2 - 6c^2g^2h^2(2g + h) - 3g^2h^4. \end{aligned}$$

$$\begin{aligned} \therefore 6(g^2 + gh + h^2)x^2 - [c^2(2g^2 - gh + 2h^2) - h^2(g + 2h)] \\ = \pm gh\sqrt{9c^4 - 6c^2(2g + h) - 3h^2}. \end{aligned}$$